

# Spatial Data Mining: Progress and Challenges

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**Abstract:** Spatial data mining, i.e., mining knowledge from large amounts of spatial data, is a highly demanding field because huge amounts of spatial data have been collected in various applications, ranging from remote sensing, to geographical information systems (GIS), computer cartography, environmental assessment and planning, etc. The collected data for exceeded human's ability to analyze. Recent studies on data mining have extended the scope of data mining from relational and transactional databases to spatial databases. This paper summarizes recent works on spatial data mining, from spatial data generalization, to spatial data clustering, mining spatial association rules, etc. It shows that spatial data mining is a promising field, with fruitful research results and many challenging issues.

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## 1. INTRODUCTION

Advances in database technologies and data collection techniques including barcode reading, remote sensing, satellite telemetry, etc., have collected huge amounts of data in large databases. This explosively growing data creates the necessity of knowledge/information discovery from data, which leads to a promising emerging field, called data mining or knowledge discovery in databases (KDD). Knowledge discovery in databases can be defined as the discovery of interesting, implicit, and previously unknown knowledge from large databases. Data mining represents the integration of several fields, including machine learning, database systems, data visualization, statistics, and information theory. Although there have been many studies of data mining in relational and transaction databases, data mining is in great demand in other applicative databases, including spatial databases, temporal databases, object-oriented databases, multimedia databases, etc. Our focus of this overview is on the methods of spatial data mining, i.e., discovery of interesting knowledge from spatial data. Spatial data are the data related to objects that occupy space. A spatial database stores spatial objects represented by spatial data types and spatial relationships among such objects. Spatial data carries topological and/or distance information and it is often organized by spatial indexing structures and accessed by spatial access methods. These distinct features of a spatial database pose challenges and bring opportunities for mining information from spatial data. Spatial data mining, or knowledge discovery in spatial database, refers to the extraction of implicit knowledge, spatial relations, or other patterns not explicitly stored in spatial databases.

Previous works in machine learning, database systems and statistics laid the foundation for research into knowledge discovery in databases. Also, advances in spatial databases, such as spatial data structures, spatial reasoning, computational geometry, etc., paved the way for the study of spatial data mining. A crucial challenge to spatial data mining is the efficiency of spatial data mining algorithms due to the huge amount of spatial data and the complexity of spatial data type and spatial accessing methods.

Spatial data mining methods can be applied to extract interesting and regular knowledge from large spatial databases. In particular, they can be used for understanding spatial data, discovering relationships between spatial and non-spatial data, construction of spatial knowledge-bases, query optimization, data reorganization in spatial databases, capturing the general characteristics in simple and concise manner, etc. This has wide applications in Geographic Information Systems (GIS), remote sensing, image databases exploration, medical imaging, robot navigation, and other areas where spatial data are used. Knowledge discovered from spatial data can be of various forms, like characteristic and discriminant rules, extraction and description of prominent structures or clusters, spatial associations, and others. The purpose of this survey

is to provide an overall picture of the methods of spatial data mining, their strengths and weaknesses, how and when to apply them, and to determine what was achieved so far and what are the challenges yet to be faced.

### 1.1 Spatial Data Mining Background

Statistical spatial analysis has been the most common approach for analyzing spatial data. Statistical analysis is a well studied area and therefore there exist a large number of algorithms including various optimization techniques. It handles very well numerical data and usually comes up with realistic models of spatial phenomena. The major disadvantage of this approach is the assumption of statistical independence among the spatially distributed data. This causes problems as many spatial data are in fact interrelated, i.e., spatial objects are influenced by their neighboring objects. Regression models with spatially lagged forms of the dependent variables can be used to alleviate this problem to some extent. Unfortunately, it makes the whole modelling process more complicated and can only be done by experts with a fair amount of domain knowledge and statistical expertise. In other words, it is not the kind of technique that we want to present to the end users for the analysis of spatial data. Furthermore, the statistical approach cannot model nonlinear rules very well and symbolic values like names are handled poorly. Statistical methods also do not work well with incomplete or inconclusive data. Another problem related to statistical spatial analysis is the expensive computation of the results.

With the advent of data mining, researchers proposed various methods for discovering knowledge from large databases. Most of them concentrate on relational or transaction databases. These methods strived to combine the already mature areas like machine learning, databases and statistics. Studies like laid a foundation for spatial data mining. Machine learning techniques learning from examples and generalization and specialization are widely used in spatial data mining. It did not take long before the statistical cluster analysis technique was modified for the use in spatial data mining . Also other methods were extended toward knowledge discovery in spatial databases. In the next section, we de ne some commonly used terms in spatial data mining.

**Rules:** Various kinds of rules can be discovered from databases in general. For example, characteristic rules, discriminant rules, association rules, or deviation and evolution rules can be mined. A spatial characteristic rule is a general description of spatial data. For example, a rule describing the general price range of houses in various geographic regions in a city is a spatial characteristic rule. A spatial discriminant rule is a general description of the features discriminating or contrasting a class of spatial data from other class(es) like the comparison of price ranges of houses in different geographical regions. Finally, a spatial association rule is a rule which describes the implication of one or a set of features by another set of features in spatial databases. For example, a rule associating the price range of the houses with nearby spatial features, like beaches, is a spatial association rule.

**Thematic maps:** Thematic maps present the spatial distribution of a single or a few attributes. This differs from general or reference maps where the main objective is to present the position of objects in relation to other spatial objects. Thematic maps may be used for discovering different rules. For example, we may want to look at temperature thematic map while analyzing the general weather pattern of a geographic region. There are two ways to represent thematic maps: raster and vector. In the raster image form thematic maps have pixels associated with the attribute values. For example, a map may have the altitude of the spatial objects coded as the intensity of the pixel (or the color). In the vector representation, a spatial object is represented by its geometry, most commonly being the boundary representation along with the thematic attributes. For example, a park may be represented by the boundary points and corresponding elevation value.

**Image databases:** These are special kind of spatial databases where data almost entirely consists of images or pictures. Image databases are used in remote sensing, medical imaging, etc. They are usually stored in form of grid arrays representing the image intensity in one or more spectral ranges.

#### 1.1.1 Spatial Data Structures, Computations, and Queries

Algorithms for spatial data mining involve the use of spatial operations like spatial joins, map overlays, nearest neighbor queries and others. Therefore, efficient spatial access methods (SAM) and data structures for such computation is also a concern in spatial data mining. We will brie y introduce some of the prominent spatial data structures and spatial computations.

**Spatial Data Structures:** Spatial data structure consists of points, lines, rectangles, etc. In order to build indices for these data, multidimensional trees have been proposed. These include quad trees, k-d trees, R-trees, R\*-trees, etc. One of the prominent SAMs which was much discussed in the literature recently is R-tree and its modification R\*-tree. Objects stored in R-trees are approximated by Minimum Bounding Rectangles (MBR). R-tree in every node stores a set of rectangles. At the leaves there are stored pointers to representation of polygon's boundaries and polygon's MBRs. At the internal nodes each rectangle is associated with a pointer to a child and represents minimum bounding rectangle of all rectangles stored in the child.

**Spatial Computations:** Spatial join is one of the most expensive spatial operations. In order to make spatial queries efficient spatial join has to be efficient as well. Brinkho et al. proposed an efficient multilevel processing of spatial joins using R\*-Trees and various approximation of spatial objects. The first step filter finds possible pairs of intersecting objects using first their MBRs and later other approximations. In the second step refinement detailed geometric procedure is performed to check for intersection. Another important spatial operation, map overlay, is especially important in Geographic Information Systems.

**Spatial Query Processing:** Optimization strategies for spatial query processing are outlined in Aref and Samet. The authors proposed an architecture for spatial database called SAND (spatial and nonspatial data) architecture, which is a model of the extended relational database with spatial operations. This architecture provides both spatial and nonspatial components of spatial database to participate in query processing and optimization.

## 1.2 Spatial Data Mining Architecture

Various architectures (models) have been proposed for data mining. They include Han's architecture for general data mining prototype DBLEARN/DBMINER, Holsheimer et al's parallel architecture, and Matheus et al.'s multicomponent architecture. Almost all of these architectures have been used or extended to handle spatial data mining. Matheus et al.'s architecture seems to be very general and has been used by other researchers in spatial data mining, including Ester et al. This architecture comparable to others is presented in Figure 1. In this architecture, the user may control every step of the mining process. Background knowledge, like spatial and non-spatial concept hierarchies, or information about database, is stored in a knowledge base. Data is fetched from the storage using the DB interface which enables optimization of the queries. Spatial data index structures, like R-trees, may be used for efficient processing. The Focusing Component decides which parts of data are useful for pattern recognition. For example, it may decide that only some attributes are relevant to the knowledge discovery task, or it may extract objects whose usage promises good results. Rules and patterns are discovered by the Pattern Extraction module. This module may use statistical, machine learning, and data mining techniques in conjunction with computational geometry algorithms to perform the task of finding rules and relations. The interestingness and significance of these patterns is then processed by Evaluation module to possibly eliminate obvious and redundant knowledge. The four last components may interact between themselves through the Controller part.

## 1.3 Organization of the paper

The rest of the paper is organized as follows. In Section 2 we survey the methods for spatial data mining, we categorize the methods and discuss each in detail. Section 2.1 describes generalization based methods, Section 2.2 discusses clustering based methods, Section 2.3 presents the methods used to explore spatial associations, Section 2.4 describes pattern recognition methods, and finally in Section 2.5 other interesting methods are outlined. We present suggestions and future directions in Section 3, and we conclude our discussion in Section 4.

## 2. METHODS FOR KNOWLEDGE DISCOVERY IN SPATIAL DATABASES

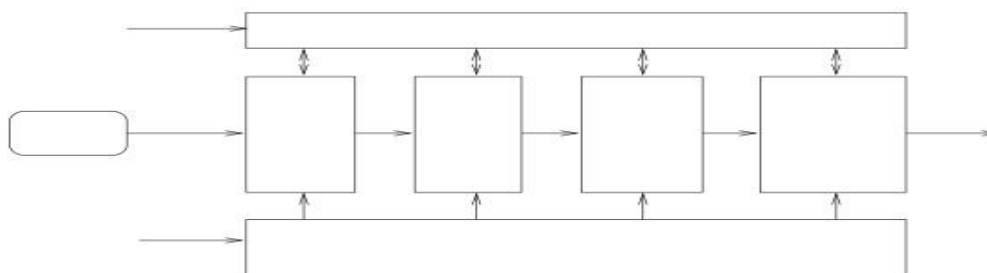
Geographic data consist of spatial objects and nonspatial description of these objects. Non-spatial description of spatial objects can be stored in a traditional relational database where one attribute is a pointer to spatial description of the object. Spatial data can be described using two different properties, geometric and topological. For example, geometric properties can be spatial location, area, perimeter, etc., whereas topological properties can be adjacency (object A is neighbor of object B), inclusion (object A is inside in object B), and others. Thus, the methods for discovering knowledge can be focused on the non-spatial and/or spatial properties of spatial objects.

The algorithms for spatial data mining include generalization-based methods for mining spatial characteristic and discriminant rules, two-step spatial computation technique for mining spatial association rules, aggregate proximity

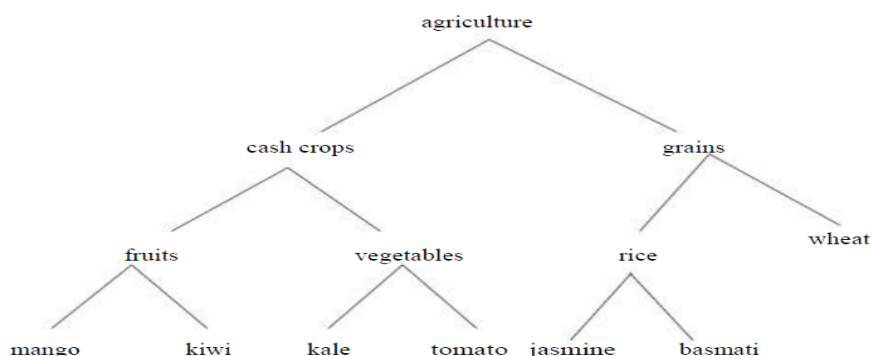
technique for finding characteristics of spatial clusters, etc. In the following sections, we categorize and describe a number of these algorithms.

## 2.1 Generalization-Based Knowledge Discovery

One of the widely used techniques in machine learning is learning from examples. This method is often combined with generalization. This approach cannot be directly adopted for large spatial databases because: 1) the algorithms are exponential in the number of examples, and 2) it does not handle noise and inconsistent data very well. Han et al. modified these techniques and gave an attribute-oriented (as opposed to the tuple-oriented in machine learning algorithms) induction algorithm to mine knowledge from large relational databases. Later Lu et al. extended this technique to spatial databases. Thus, the assumptions that are made for relational databases are also carried to spatial data mining. The generalization-based knowledge discovery requires the existence of background knowledge in the form of concept hierarchies.



**Figure 1:** An architecture for a KDD system



**Figure 2:** Example of agricultural land use concept hierarchy

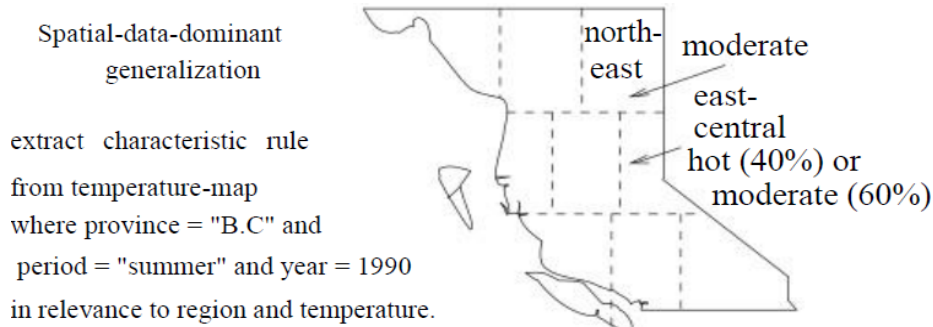
In the case of spatial databases, there can be two kinds of concept hierarchies, non-spatial and spatial. Concept hierarchies can be explicitly given by the experts, or in some cases they can be generated automatically by data analysis. An example of a concept hierarchy for agricultural land use is shown in Figure 2. As we ascend the concept tree, information becomes more and more general, but still remains consistent with the lower concept levels. For example, in Figure 2 both jasmine and basmati can be generalized to the concept rice which in turn can be generalized to concept grains, which also includes wheat. A similar hierarchy may exist for spatial data. For example, in a generalization process, regions representing counties can be merged to provinces and provinces can be merged to larger regions. Attribute-oriented induction is performed by climbing the generalization hierarchies and summarizing the general relationships between spatial and non-spatial data at higher concept levels. It can be done on non-spatial data by (a) climbing the concept hierarchy when attribute values in a tuple are changed to the generalized values, (b) removing attributes when further generalization is impossible and there are too many different values for an attribute, and (c) merging identical tuples. Induction is continued until every attribute is generalized to the desired level. The desired level is reached when the number of different values for the attribute in the generalized table is no greater than the generalization threshold for this attribute.

During the process of merging of identical tuples the number of merged tuples is stored in additional attribute count to enable quantitative presentation of acquired knowledge. Lu et al. presented two generalization based algorithms, spatial-data-dominant and non-spatial-data-dominant generalizations. Both algorithms assume that the rules to be mined are general data characteristics and that the discovery process is initiated by the user who provides a learning request (query) explicitly, in a syntax similar to SQL. We will briefly describe both algorithms as follows:

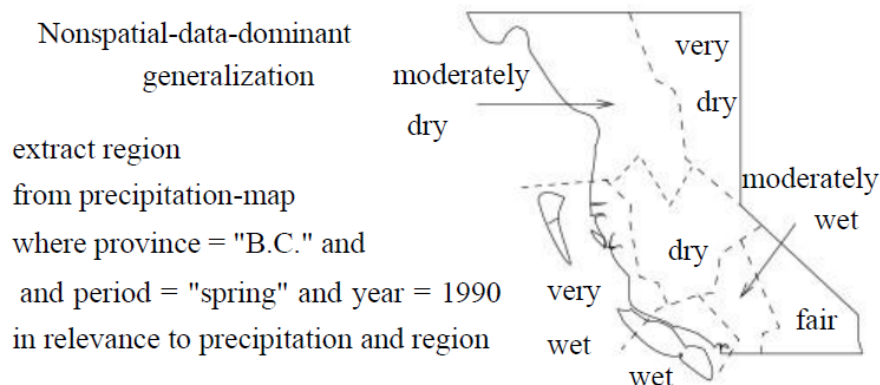
**Spatial-Data-Dominant Generalization:** In the first step all data described in the query are collected. Given the spatial data hierarchy, generalization can be performed first on the spatial data by merging the spatial regions according to the description stored in the concept hierarchy. Generalization of the spatial objects continues until the spatial generalization threshold is reached. The spatial generalization threshold is reached when the number of regions is no greater than the threshold value. After the spatial-oriented induction process, non-spatial data are retrieved and analyzed for each of the spatial objects using the attribute oriented induction technique as described above. An example of a query and the result of the execution of the spatial-data-dominant generalization algorithm is presented in Figure 3. In this example, temperature in the range [20, 27] is generalized to moderate, and temperature in the range [27, 1) to hot. The answer to the query is the description of all regions using a disjunction of a few predicates which characterize each of the generalized regions. Temperature measured in the east-central region of British Columbia is in the range [22, 30]. Thus, in our example, the description of the temperature weather pattern in this region is hot or moderate. The computational complexity of the algorithm is  $O(N \log N)$ , where  $N$  is the number of spatial objects.

**Non-spatial-Data-Dominant Generalization:** This method also starts with collecting all data relevant to the user query. In the example presented in Figure 4 the DB interface extracts the precipitation data relevant to the province and time period specified in the query. In the second step the algorithm performs attribute oriented induction on the non-spatial attributes, generalizing them to a higher (more general) concept level. For example, the precipitation value in the range (10 in., 15 in.] can be generalized to the concept wet. The generalization threshold is used to determine whether to continue or stop the generalization process. In this step the pointers to spatial objects are collected as a set and put with the generalized non-spatial data. In the third and the last step of the algorithm, neighboring areas with the same generalized attributes are merged together based on the spatial function adjacent to. For example, if in one area the precipitation value was 17 in., and in neighboring area it was 18 in. both precipitation values are generalized to the concept very wet and both areas are merged. Approximation can be used to ignore small regions with different non-spatial description. For example, if the majority of area land can be described as industrial, but a few gas stations exist in this area the whole area can be described as industrial one. The result of the query may be presented in the form of a map with a small number of regions with high level descriptions as it is shown in Figure 4. The computational complexity of this algorithm is also  $O(N \log N)$ , where  $N$  is the number of spatial objects.

We presented two generalization based algorithms that assumed the concept hierarchies to be given or generated automatically. However, as pointed out before, there may be cases where such hierarchies are not present a priori. Another problem with previous algorithms is that the spatial components of the databases are explored by merging regions at lower levels of the concept hierarchy to form region(s) at higher levels of the hierarchy. Both of these facts suggest that the quality and the interestingness of the mined characteristic rules is going to be much dependent upon the given concept hierarchy(ies). In many cases such hierarchies are given by the experts and they may be not entirely appropriate. Therefore, we would like to find algorithms that do not need to use these hierarchies. We will describe an algorithm not depending on spatial concept hierarchies in the next section.



**Figure 3:** Example of a query and the result of the execution of the spatial-data-dominant generalization method



**Figure 4:** Example of a query and the result of the execution of the non-spatial-data-dominant generalization method

## 2.2 Methods Using Clustering

Cluster analysis is a branch of statistics that has been studied extensively for many years. The main advantage of using this technique is that interesting structures or clusters can be found directly from the data without using any background knowledge, like concept hierarchies. A similar approach in machine learning is known as unsupervised learning. We can exploit the results of research on clustering techniques in the spatial data mining process as proposed in.

Clustering algorithms used in statistics, like PAM or CLARA, are reported to be inefficient from the computational complexity point of view. As for the efficiency concern, a new algorithm, called CLARANS (Clustering large Applications based upon RANdomized Search), was developed for cluster analysis. Experimental evidence showed that CLARANS outperforms the two existing cluster analysis algorithms, PAM (Partitioning Around Medoids) and CLARA (Clustering LARge Applications). Ng and Han used CLARANS in spatial data mining algorithms, SD(CLARANS) and NSD(CLARANS). First, we will briefly describe the three cluster analysis algorithms.

The PAM algorithm was developed by Kaufman and Rousseeuw. Assuming that there are  $n$  objects, PAM finds  $k$  clusters by understanding a representative object for each cluster. Such a representative, which is the most centrally located point in a cluster, is called a medoid. After selecting  $k$  medoids, the algorithm repeatedly tries to make a better choice of medoids analyzing all possible pairs of objects such that one object is a medoid and the other is not. The measure of clustering quality is calculated for each such combination. The best choice of points in one iteration is chosen as the medoids for the next iteration. The cost of a single iteration is  $O(k(n-k)^2)$ . It is therefore computationally quite inefficient for large values of  $n$  and  $k$ .

The CLARA algorithm was proposed by Kaufman and Rousseeuw as well. The difference between the PAM and CLARA algorithms is that the latter one is based upon sampling. Only a small portion of the real data is chosen as a representative of the data and medoids are chosen from this sample using PAM. The idea is that if the sample is selected in a fairly random manner, then it correctly represents the whole data set and therefore, the representative objects (medoids) chosen, will be similar as if chosen from the whole data set. CLARA draws multiple samples and outputs the best clustering out of these samples. As expected, CLARA can deal with larger data sets than PAM. The complexity of each iteration now becomes  $O(kS^2 + k(n-k))$ , where  $S$  is the size of the sample. The authors indicated through their experimental results that samples of size  $40+2k$  give good results.

It is easy to realize that PAM searches for the best  $k$  medoids among a given data set whereas CLARA searches for the best  $k$  medoids among the selected sample of the data set. Let us suppose that object  $O_i$  is one of the medoids in the best  $k$  medoids. Thus, if during sampling  $O_i$  is not selected, then CLARA will never find the best clustering. This is exactly the tradeoff for efficiency. Ng and Han's [41] proposed CLARANS algorithm which tries to mix both PAM and CLARA by searching only the subset of the data set and it does not confine itself to any sample at any given time. While CLARA has a fixed sample at every stage of the search, CLARANS draws a sample with some randomness in each step of the search. The clustering process can be presented as searching a graph where every node is a potential solution, i.e., a set of  $k$  medoids. The clustering obtained after replacing a single medoid is called the neighbor of the current clustering. The number of neighbors to be randomly tried is restricted by the parameter max neighbor. If a better neighbor is found CLARANS moves to the neighbor's node and the process is started again, otherwise the current clustering produces a

local optimum. If the local optimum is found CLARANS starts with new randomly selected node in search for a new local optimum. The number of local optima to be searched is also bounded by the parameter num local. CLARANS has been experimentally shown to be more efficient than both PAM and CLARA. The authors claim that the computational complexity of every iteration in CLARANS is basically linearly proportional to the number of objects. This claim has been supported by Ester et al. in. It should be mentioned that CLARANS can be used to find the most natural number of clusters  $k_{nat}$ . The authors adopted a heuristic of determining  $k_{nat}$ , which uses silhouette coefficients<sup>1</sup>, introduced by Kaufman and Rousseeuw [31]. CLARANS also enables the detection of outliers, e.g., points that do not belong to any cluster.

Based upon CLARANS, two spatial data mining algorithms were developed in a fashion similar to the algorithms discussed earlier in this section: spatial dominant approach, SD(CLARANS) and non-spatial dominant approach, NSD(CLARANS). Both algorithms assume that the user specifies the type of the rule to be mined and relevant data through a learning request in a similar way as in the experimental database mining prototype, DBLearn.

#### Algorithm SD (CLARANS)

In this spatial dominant approach, spatial component(s) of the relevant data items are collected and clustered using CLARANS. Then, the algorithm performs an attribute-oriented induction on non-spatial description of objects in each cluster. The result of the query presents high-level non-spatial description of objects in every cluster. For example, one can find that in Vancouver expensive housing units are clustered in 3 clusters. In the downtown cluster there are mainly expensive condominiums; in the waterfront cluster mansions and single houses are located; and the third cluster consists mainly of single houses.

#### Algorithm NSD (CLARANS)

This non-spatial dominant approach first applies non-spatial generalizations. Attribute-oriented generalization is performed on the non-spatial attributes and produces a number of generalized tuples. For example, the descriptions of expensive housing units can be generalized to single houses, mansions and condominiums. Then, for each such generalized tuple, all spatial components are collected and clustered using CLARANS to find  $k_{nat}$  clusters. In the final step, the clusters obtained that way are checked to see if they overlap with clusters describing other types of objects. If so, then the clusters are merged, and the corresponding generalized non-spatial descriptions of tuples are merged as well.

Depending upon the rules or the form of knowledge that user wants to discover, it may be better to choose one or the other of the above two algorithms. Usually SD (CLARANS) is more efficient than NSD(CLARANS). But, when the distribution of points is mainly determined by their non-spatial attributes NSD(CLARANS) may have an edge.

### CLARANS in large Spatial Databases

#### Focusing Methods:

Ester et al. pointed out some of the drawbacks of the CLARANS clustering algorithm [41]. First of all, CLARANS assumes that the objects to be clustered are all stored in main memory. This assumption may not be valid for large databases and that is why disk-based methods could be required. Secondly, the efficiency of the algorithm can be substantially improved by modifying the focusing component of the algorithm (see architecture in Figure 1).

The first drawback is alleviated by integrating CLARANS with efficient spatial access methods, like  $R^*$ -tree, and tree.  $R^*$ -tree supports the focusing techniques that Ester et al. proposed to reduce the cost of computations. It showed that the most computationally expensive step of CLARANS is calculating the total distances between the two clusterings. Thus, the authors proposed two approaches to reduce the cost of this step. The first one is to reduce the number of objects to consider. A centroid query returns the most central object of a leaf node of the  $R^*$ -tree where neighboring points are stored. Only these objects are used to compute the medoids of the clusters. Thus, the number of objects taken for consideration is reduced. This technique is called focusing on representative objects.

The drawback is that some objects, which may be better medoids, are not considered, but the sample is drawn in the way which still ensures good quality of clustering. The other technique to reduce the computations is to restrict the access to certain objects that do not actually contribute to the computation. The authors further gave two different

focusing techniques which try to exploit this approach: focus on relevant clusters, and focus on a cluster. Using R\*-tree structure the authors proposed a way of performing computation only on pairs of objects that can improve the quality of clustering instead of checking all pairs of objects as it is done in CLARANS algorithm.

Ester et al. Applied the focusing on representative objects to a large protein database to find the segmentation of protein surfaces so as to facilitate the so-called docking queries. They reported that when the focusing technique was used the effectiveness decreased just from 1.5% to 3.2% whereas the efficiency increased by factor 50, which was the number of points stored in a disk page. The measure of effectiveness used is the average distance of the resulting clustering whereas the measure of efficiency used is the CPU time.

### Clustering Features and CF trees

R-trees are not always available and their construction may be time consuming. Z [52] pre-hang et. al. presented another algorithm BIRCH (Balanced Iterative Reducing and Clustering) for clustering of large sets of points. The method they presented is the incremental one with possibility of adjustment of memory requirements to the size of memory that is available. The authors used concepts called Clustering Feature and CF tree.

A Clustering Feature CF is the triple summarizing information about subclusters of points. Given Nd-dimensional points in the subcluster: fXig, CF is defined as.

$$CF = (N; L \sim S; SS)$$

where N is the number of points in the subcluster, PNLs is the linear sum on N points, i.e.,  $\sum_{i=1}^N X_i$

SS is the square sum of data points, i.e.,  $\sum_{i=1}^N X_i^2$

The Clustering Features are sufficient for computing clusters and they constitute an efficient information storage method as they summarize information about the subclusters of points instead of storing all points.

**A CF tree is a balanced tree with two parameters:** branching factor B and threshold T. The branching factor species maximum number of children. The threshold parameter species the maximum diameter of subclusters stored at the leaf nodes. By changing the threshold value we can change the size of the tree. The non-leaf nodes store sums of their children's CFs, and thus, they summarize the information about their children. The CF tree is build dynamically as data points are inserted. Thus, the method is an incremental one. A point is inserted to the closest leaf entry (subcluster). If the diameter of the subcluster stored in the leaf node after insertion is larger than the threshold value, then, the leaf node and possibly other nodes are split. After the insertion of the new point the information about it is passed towards the root of the tree. One can change the size of the CF tree by changing the threshold. If the size of the memory that is needed for storing the CF tree is larger than the size of the main memory, then a larger value of threshold is specied and the CF tree is rebuilt. The rebuild process is performed by building a new tree from the leaf nodes of the old tree. Thus, the process of rebuilding the tree is done without the necessity of reading all the points. Therefore, for building the tree data has to be read just once. The authors present also some heuristics for dealing with outliers and methods for improving the quality of CF trees by additional scans of the data.

Zhang et. al. claim that any clustering algorithm, including CLARANS may be used with CF trees. The CPU and I/O costs of the BIRCH algorithm are  $O(N)$ . The authors performed a number of experiments which showed linear scalability of the algorithm with respect to number of points, insensibility to the input order, and good quality of clustering of the data.

### 2.3 Methods Exploring Spatial Associations

All methods that we discussed in previous sections find only characteristic rules that characterize spatial objects according to their nonspatial attributes. In many situations we want to discover spatial association rules, rules that associate one or more spatial objects with other spatial objects. The concept of association rules was introduced by Agrawal et al. [1] in a study of mining large transaction databases. Koperski and Han [34] extended this concept to spatial databases.



A spatial association rule is of the form  $X \rightarrow Y (c\%)$ , where  $X$  and  $Y$  are sets of spatial or non-spatial predicates and  $c\%$  is the confidence of the rule. For example, the following rule is a spatial association rule:  $\text{is a}(x, \text{school}) \rightarrow \text{close to}(x, \text{park}) (80\%)$ . This rule states that 80% of schools are close to parks. There are various kinds of spatial predicates that could constitute a spatial association rule. Some examples are: topological relations like intersects, overlap, disjoint, etc.; spatial orientations like left of, west of, etc.; distance information, such as close to, far away, etc.

To confine the number of discovered rules, the concepts of minimum support and minimum confidence are used. The intuition behind this is that in large databases, there may exist a large number of associations between objects but most of them will be applicable to only a small number of objects, or the confidence of rules may be low. For example, the user may not be interested in the relation associating 5% of houses and a single school. He/she may be interested in rules that apply to at least 50% of houses. We would like to filter out associations describing small percentage of objects using the minimum support thresholds. We also want to filter out rules with low confidence using minimum confidence threshold. These thresholds can be different at each level of non-spatial description of objects since

the same thresholds may not find interesting associations at the lower concept levels where the number of objects having the same description is smaller. Thus, at the lower levels of non-spatial hierarchies the percentage of objects may not reach the support threshold for the higher levels. Informally, we can define the support of a pattern  $A$  in a set  $S$  to be the likelihood of the occurrence of pattern  $A$  in  $S$ , and the confidence of rule  $X \rightarrow Y$  to be the likelihood that the pattern  $Y$  for object  $O$ s occurs whenever  $X$  occurs for the same object. A set of predicates  $P$  is large in set  $S$  at level  $l$  of the non-spatial concept hierarchy if the support of  $P$  is no less than its minimum support threshold  $\theta$  for level  $l$  (it is true for large number of objects), and all ancestors of  $P$  from the concept hierarchy are large at their corresponding levels. A strong rule is a rule with large support, i.e., no less than the minimum support threshold, and large confidence, i.e., no less than the minimum confidence threshold. A top-down, progressive deepening search method for mining strong spatial association rules is described in [34]. To minimize the number of costly spatial computations a novel two step spatial computation technique for optimization during the search for associations was introduced. Computation starts at the high level of spatial predicates like  $g$  close to (generalized close to). A pair of objects satisfies the predicate  $g$  close to if their Minimum Bounding Rectangles are located in the distance no greater than the threshold for this predicate.

Thus, we deal with the problem of the intersections of isothetic rectangles. Efficient spatial computation algorithms and structures like R-trees or plane-sweep techniques can be used in this step. More detailed and finer, but more expensive, spatial computations are applied at lower concept levels only to those patterns that are large at the level of the predicate  $g$  close to. The rationale behind this is that if a pattern is not large at  $g$  close to level it certainly will not be large at the level of detailed spatial relations. Filtration of large patterns saves a great deal of computations since there are much fewer spatial association relationships left at the lower concept levels. The filtration process is done using minimum support at the high levels.

#### Algorithm for Multiple Level Spatial Association Rules

The mining process is started by a query which is to describe a class of objects  $S$  using other task relevant classes of objects, and a set of relevant relations. For example, a user may want to describe parks by presenting the description of relations between parks and other objects like: railways, restaurants, zoos, hydrological objects, recreational objects, and roads. Furthermore, the user can state that he/she is interested only in objects in the distance less than one kilometer from a park. The first step of the algorithm collects the task-relevant data. Then, some efficient spatial computations are performed as mentioned above to extract spatial associations at the level of generalized spatial relations. These efficient computations look for objects whose minimal bounding rectangles are located in the distance no greater than the threshold to satisfy the close to predicate. In this way, objects satisfying the predicate  $g$  close to (generalized close to) are found. This predicate encompasses exact spatial predicates like adjacent to, intersects, distance less than  $x$ . The  $g$  close to predicates are stored in an extended relational database Coarse predicate DB. Every row of the Coarse predicate DB is a description of a single object from the class of objects being described. Description consists of objects which satisfy task relevant predicates. For example, a row related to Stanley Park in Vancouver may include restaurant, zoo, main road, inlet, lake and other objects located inside the park or close to it. Each predicate in Coarse predicate DB is checked with the threshold for the top level to filter out task-relevant classes of objects in the  $g$  close to predicates which do not promise getting large predicates. For example, if only 5% of objects from class  $S$  satisfy

the predicate  $g$  close to  $(s, zoo)$  and the minimum support threshold on the top level is 15% then the predicates  $g$  close to  $(s, zoo)$  will be deleted. This filtration results in a database of large predicates (Large Coarse predicate DB). A spatial association rules at the coarse level can be generated from Large Coarse predicate DB. This database is further processed using finer spatial computations to produce Fine predicate DB. In the Fine predicate DB, generalized predicates like  $g$  close to are changed into exact spatial predicates like adjacent to, intersects, or on the top level is 15% then the predicates  $g$  close to  $(s, zoo)$  will be deleted. This filtration results in a database of large predicates (Large Coarse predicate DB). A spatial association rules at the coarse level can be generated from Large Coarse predicate DB. This database is further processed using finer spatial computations to produce Fine predicate DB. In the Fine predicate DB, generalized predicates like  $g$  close to are changed into exact spatial predicates like adjacent to, intersects, or distance less than  $x$ . We call a single predicate, like close to  $(x, lake)$ , a 1-predicate. The conjunction of  $k$  such predicates is called a  $k$ -predicate. For example, the predicate  $close\ to(x, lake) \wedge close\ to(x, restaurant)$  is a 2-predicate. This predicate states that the object  $x$  is both close to a lake and close to a restaurant. The Fine predicate DB is used to produce large  $k$ -predicates and generate association rules at multiple concept levels. At each concept level, the algorithm starts with large 1-predicates and iteratively generates large  $k$ -predicates until no large  $(k+1)$ -predicate can be found by adding a large 1-predicate to any large  $k$ -predicate. The algorithm finds large predicates by counting the number of occurrences of predicates in the database and comparing this number with the support threshold. The predicates and the number of their occurrences in Fine predicate DB are stored in the predicate table. Based on the information stored in the predicate table the algorithm derives strong rules. For example, if the predicate  $close\ to(x, lake)$  occurs in 100 rows of Fine predicate DB, the predicate  $close\ to(x, restaurant)$  occurs in 90 rows, and both predicates  $close\ to(x, lake)$  and  $close\ to(x, restaurant)$  occur together in 80 rows, then the rule  $\{is\ a(x, park) \wedge close\ to(x, lake) \} \rightarrow close\ to(x, restaurant) (80\%)$  may be derived. After finding large predicates on high levels of concept hierarchies, the algorithm tries to enlarge predicates and rules on lower levels. For example, restaurants may be specialized into oriental restaurants and continental restaurants, and the algorithm may find relations between parks and these types of restaurants.

The computational complexity of the algorithm is  $O(C_c n_c + C_f n_f + C_{nonspatial})$  [34], where  $C_c$  and  $C_f$  are average costs of computing each spatial predicate at a coarse and fine resolution level respectively,  $n_c$  is the number of predicates that are coarsely computed,  $n_f$  is the number of predicates that are finely computed, and  $C_{nonspatial}$  is the total cost of generating rules from the predicate databases. It is observed that  $n_f$  is smaller than  $n_c$ , but  $C_c$  is more efficient than  $C_f$ .

The above algorithm, especially the two-step computation technique, is a novel approach towards mining spatial association rules at multiple levels. It requires background knowledge in the form of concept hierarchies and expects a user to describe the form of the rule s/he wants by giving such information in the mining query. It may be a good idea to work towards integration of this technique with clustering methods to avoid the necessity of the user having to provide the concept hierarchies for spatial and nonspatial attributes.

## 2.4 Using Approximation and Aggregation

We discussed a clustering algorithm CLARANS in Section 2.2. The algorithm is an effective and efficient method of finding where the clusters in the spatial database are, i.e., partitioning data into clusters. However, perhaps the more interesting result would be to find out why the clusters are there. Knorr and Ng in [10] presented a study motivated by this question. This question can be rephrased as "what are the characteristics of the clusters in terms of the features that are close to them". The problem is how to measure the aggregate proximity, because statements like 90% of the houses in a cluster are close to the feature  $F$  are more informative and interesting than statements like one house is close to a certain feature  $F$ . The aggregate proximity is the measure of closeness of the set of points in the cluster to a feature as opposed to the distance between a cluster boundary and the boundary of a feature.

One may ask why the authors are not simply using the  $k$  nearest neighbor searches using structures like  $k$ -d trees,  $R$ -trees and its variants, Voronoi diagrams, etc. It turns out that such structures are unable to perform the search needed for their purpose. For example, the distance between the cluster and a feature is measured as the distance between the boundaries, not between the points, like centroids. Furthermore, the costs of building and maintaining the indices are prohibitive given the fact that such indices may not be used frequently. Therefore, the authors propose the use of computational

geometry concepts to find out the characteristics of a given cluster in terms of the features close to it. The authors described the algorithm CRH (where C is for encompassing circle, R for isothetic rectangle, and H for convex hull) which uses such concepts as filters to reduce the candidate features at multiple levels. In short, they collect a large number of features from multiple maps and feed them along with the cluster to the algorithm CRH and discover knowledge about spatial relationships as shown in Figure 5.

### Algorithm CRH

Knorr and Ng evaluated various computational geometry algorithms for distance computation, and shape descriptions and overlap computations. Taking into account the problem of data distribution in a cluster and various sizes and shapes of the features, the authors chose a technique for computing the distance between a cluster point and feature boundary. For the shape description, the authors chose minimum bounding structures. They used these structures to develop a multiple filtering approach, with the filters set up in an increasing order of accuracy but decreasing order of efficiency.

That is, filters that are applied earlier are more efficient but coarser than the later ones.

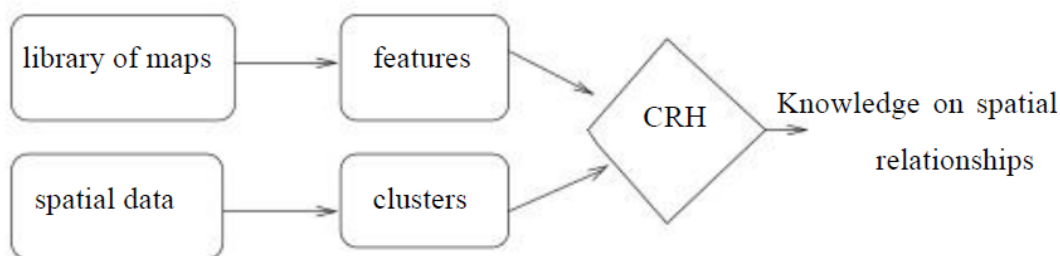
The algorithm CRH first applies the encompassing circle filter to the large number of features. Features that are the most promising ones are passed to the isothetic rectangle filter. These two filters eliminate a large number of features and only a small number of features is passed to the final convex hull filter. Then, the CRH algorithm calculates the aggregate proximity of points in the cluster to the convex boundary of each feature, upon which the features are ranked. Also, each filter has its own threshold, which is the minimum number of features to pass on to the next filter. When the number of features found lies below the threshold, the cluster is enlarged to encompass more features to pass the threshold limit. Shape enlargement can be achieved by the linear policy (enlarge the shape by constant distance), or by the bisection policy. Bisection policy performs enlargement or diminution of the area by a distance which decreases logarithmically. This policy checks if enough features are in the area of shape and enlarges or decreases the area according to the need.

The problem with this method is that a feature may have to be tested for overlap with a cluster many times. The technique, which is called by the authors mem-oization, can be used to avoid multiple computations by storing the distance between each feature and the cluster the first time the intersection test is performed. Depending upon the shapes, circles, rectangles or convex hulls, the minimum distance between the circumferences, the boundaries of the rectangles, or the boundaries of the polygons respectively are stored. Finally, the algorithm reports the features with the smallest aggregate proximities showing minimum and maximum distances of points in the cluster to the feature, average distance, and percentages of points located in the distance less than specified thresholds. The algorithm CRH is experimentally reported to have the response time of less than two seconds for processing 50,000 features. Furthermore, it is empirically shown to be scalable and the memorization policy is found to be the most consistent and efficient of all the shape enlargement policies.

### 2.5 Mining in Image Databases

Knowledge mining from Image Databases can be viewed as a case of spatial data mining. There have been studies, led by Fayyad et al., on the automatic recognition and categorization of astronomical objects. The authors presented a system for identifying volcanos on the surface of Venus from images transmitted by the Magellan spacecraft. The Magellan transmitted more than 30,000 high resolution synthetic aperture radar images of the surface of Venus from different angles. The system is composed of three basic components: data focusing, feature extraction, and classification learning. Like all other data focusing techniques, the first component increases the overall efficiency of the system by first identifying the portion of the image being analyzed that is most likely to contain a volcano. This is achieved by comparing the intensity of the central pixel of a region to the estimated mean background intensity of its neighborhood pixels. The second component of the system extracts interesting features from the data. Standard methods used in pattern recognition like edge detection or Hough transform, deal poorly with the variability and noise presented in the case of natural data. Since it is difficult to find attributes describing volcanos exactly, matrices containing volcanos images were decomposed to eigenvectors. Eigen values were treated as attributes describing volcanos. Then the final task, which is performed by the rest of the system, is to discriminate between volcanos and other objects looking like volcanos. Such "false alarms" are caused by objects on the surface of Venus causing intensity deviations. The final component of

the system uses training examples provided by the experts to create a classifier that can discriminate between volcanos and "false alarms". The decision tree method was used for this task. The incidence angle of the synthetic aperture radar to the planet instrument strongly influenced images of volcanos. Thus, the images were normalized according to this angle. The obtained accuracy was about 80%.



**Figure 5:** Using CRH for knowledge discovery in spatial databases

In general, it is difficult for experts to provide classifications with 100% certainty and false classifications can produce large errors during classification because they are treated as negative examples. Smyth et al. in [48] discussed such issues, using the above problem as a case study. The paper's main contribution is the modeling and treatment of subjective label information given by the experts using probabilistic models. This research is important because it concludes that it is possible for the knowledge discovery methods to be modified to handle the lack of absolute ground truths.

In another study the Second Palomar Observatory Sky Survey (POSS-II) decision tree methods were also used for the classification of galaxies, stars and other stellar objects. About 3 TB of sky images were analyzed. Data images were preprocessed by low level image processing system FOCAS, which selected objects and produced basic attributes like: magnitudes, areas, intensity, image moments, ellipticity, orientation, etc. Objects in the training data set were classified by astronomers. Based on this classification, about ten training sets for decision tree algorithm were constructed. From the trees obtained by the learning algorithm, a minimal set of robust, general and correct rules was found. If no additional attributes describing features of a single image plate were used, the accuracy was about 75%. Additional attributes were defined to reach a higher level of accuracy in every image. "Surestars" were detected in every image for the purpose of finding image resolution. To gain efficiency, this process was also automated. Using "sure-stars", two additional attributes for every image plate were computed: resolution scale and resolution fraction. These two attributes were used for normalization of attributes describing objects produced by FOCAS. Other attributes like background level or average intensity were also used to normalize plates. After the normalization the classification accuracy increased to about 94%. About 5 108 objects were classified. Obtained resolution was one magnitude better than the previous astronomical studies and it was possible to classify objects with images too faint to be classified by astronomers. The performance of decision tree methods was compared with neural networks. The tested neural networks algorithms were (a) traditional back propagation, (b) conjugate gradient optimization, and (c) variable metric optimization. The last two algorithms use numerical optimization methods to compute network weights. A number of different networks was tested. The performance was fairly unstable with accuracy varying from 30% to 95%. Additional drawback of neural networks was the requirement to specify internal parameters such as the number of hidden layers or size. For future investigation, testing of unsupervised clustering techniques is planned.

The above studies showed the problems related to differences between images. The necessity of "normalization" of plates was shown to improve intra and interplate classification.

Another example of image database mining is Stolorz et al's [49] study of fast spatio-temporal data mining from geophysical data sets. The authors described a distributed parallel querying and analysis environment called CONQUEST (CONTENT-based QUerying in Space and Time). CONQUEST can be distinguished from other image database mining tools as it takes into account also temporal components of the datasets and it is designed to take advantage of parallel and distributed processing. CONQUEST was tested on two large climate datasets<sup>6</sup> to detect cyclones and blocking features. The authors used heuristic rules based on signal processing methods for the extraction of characteristic weather

phenomena. Different task decomposition methods were used to facilitate the distribution of work among a group of machines. In the case of cyclone detection, the optimal solution was the decomposition into separate temporal slices. The decomposition in the temporal dimension is not always the best solution, especially when the state plays an important role in the detection of characteristic features. For detection of blocking features, the spatial decomposition, which assigns different blocks of grid points to different machines, was proven to be optimal. After detection of weather phenomena the authors used a clustering algorithm for the detection of shared spatial features. The goal of the authors is the building of a system that combines easy formulated queries with fast parallel execution and visualization of results for refinements of the queries.

## 2.6 Other Methods

The problem introduced by Fayyad et al.'s has been followed up by other researchers as well. One interesting study was done by Bell et al. who proposed a method for knowledge discovery in spatial databases based upon evidence theory. The authors took the image database mining problem described above as a case study. In this study the authors described an extension of general framework for database mining in relational databases based on evidential theory to mine knowledge from spatial databases.

Evidential reasoning is a generalization of conventional probability in the sense that it does not make any assumptions about the independence of data being analyzed. Therefore, the evidential reasoning may be a better choice than using probabilistic model like the Bayesian method to model the data like Venus pictures, where pixels may be interrelated. Evidential theory provides a method to combine evidences gathered from different sources to produce a single measure of uncertainty. Thus, it is claimed to be a better method to reason about spatial data in the presence of uncertainty. The combination of evidences is done using a technique based upon Dempster-Shafer theory. Informally, this theory can be regarded as a generalization of the conventional probability theory, where the probabilities are fixed and known in advance, to the case where only the upper and lower bounds on probabilities are available. Bell et al. gave an example of how this method can be applied to image databases.

Major et al. used IXLT M commercial tool for mining of the tropical storm database. The goal was to predict if hurricanes can reach the U.S. territory. Data describing hurricanes were decomposed to observations at points. These observations were stored in a traditional relational database. Attributes like position of the hurricane, speed, direction, angle to the coast, etc. were used. Since multiple tuples describing the single hurricane in different points were stored, some data were interdependent. The interdependency of data causes problems, because the algorithm which was used assumes independence of data. The best rules according to different criteria like performance, novelty, significance and simplicity were selected from rules derived by the IXL. The GIS system was used to support the selection of the best rules. This study shows the necessity of extension of traditional data mining techniques toward spatial data mining for better analysis of complex spatial phenomena and spatial objects.

## 3. FUTURE DIRECTIONS

As we mentioned earlier, data mining is a young field going back no more than the late 1980s. Spatial data mining is even younger since data mining researchers first concentrated on data mining in relational databases. Many spatial data mining methods we analyzed actually assume the presence of extended relational model for spatial databases. But it is widely believed that spatial data are not handled well by relational databases. As advanced database systems, like Object-Oriented (OO), deductive, and active databases are being developed, methods for spatial data mining should be studied in these paradigms.

Data mining in spatial object-oriented databases: How can the OO approach be used to design a spatial database [40, 42] and how can knowledge be mined from these databases? It is an important question since many researchers have pointed out that OO database may be a better choice for handling spatial data rather than traditional relational or extended relational models. For example, rectangles, polygons, and more complex spatial objects can be modeled naturally in OO database. OO database techniques are maturing. OO knowledge representation techniques for spatial data have been proposed Mohan and Kashyap, and efficient SAM, like R-trees can be used to make OO database more efficient in access

and retrieval of data. Therefore, exploiting OO technology in data mining is an area with enormous potential. Techniques for generalizations of complex data objects, methods and class hierarchies have been studied by Han et al.

**Mining under uncertainty:** Use of evidential reasoning can be explored in the mining process for image databases and other databases where uncertainty modeling has to be done. As mentioned in Bell et al's, evidential theory can model uncertainty better than traditional probabilistic models, like Bayesian methods. Fuzzy sets approach was applied to spatial reasoning and it can be extended to spatial data mining.

**A filter native clustering techniques:** Another interesting future direction is the clustering of possibly overlapping objects like polygons as opposed to the clustering of points. Clusters can also maintain additional information about each object they contain, which can be the degree of membership. In this way, fuzzy clustering techniques can be used to accommodate objects having the same distance from the medoid.

**Mining Spatial Data Deviation and Evolution Rules:** One extension of current work in spatial data mining toward spatio-temporal databases is to study data deviation and evolution rules. For example, we can find spatial characteristic evolution rules which summarizes the general characteristics of the changing data. During the mining process one can discover properties of the regions with average growth of crops over 2% per year. A spatial discriminant evolution rule discriminates the properties of objects in the target class from those in the contrasting classes. For example, one can make a comparison of the areas where air pollution increased last year with the areas where the air quality has been improved. These rules may be used, for example, in medical imaging, where one would like to find out how certain features are deviating from the norm or how they are evolving over time. Other applications may include, discovering and predicting weather patterns of geographic regions, land use planning, and others.

**Using Multiple Thematic Maps:** We discussed generalization-based methods which used a single thematic map during generalizations. Various applications demand spatial data mining to be conducted using multiple thematic maps. This would involve not only clustering but also spatial computations like map overlay, spatial joins, etc. For example, to extract general weather patterns, it may be better to use temperature and precipitation thematic maps and to carry out generalization in both.

**Interleaved generalization:** To extend the generalization-based methods, it is interesting to consider interleaving spatial and nonspatial generalizations to get the results in more efficient manner. Efficient processing can be achieved because usually spatial operations, like joins and overlays, are more expensive than nonspatial computations. Thus, by first generalizing the non-spatial component and minimally using spatial generalizations one may save a lot of computation time.

**Generalization using temporal spatial data:** This relates to the point we raised on discovery of data evolution rules earlier in this section. It may involve generalization over a sequence of maps collected during different time intervals. Then, comparison or summarization can be done to discover data evolution regularities.

**Parallel Data Mining:** Due to the high volume of spatial data used during the computations mining using parallel machines or distributed farms of workstations can accelerate significantly the work. We expect that parallel knowledge discovery will be a growing research issue in both relational and spatial data mining.

**Cooperation between Statistical Analysis and Data Mining:** The enhancement of data mining techniques with mature statistical methods may produce interesting new techniques which may work well with different kinds of problems and on different data. For example, the statistical techniques may help in judgement on interestingness and significance of rules.

**Spatial Data Mining Query Language:** Design of the user interface can be one of the key issues in the popularization of knowledge discovery techniques. One can create a query language which may be used by non-database specialists in their work. Such a query interface can be supported by Graphical User Interface (GUI) which can make the process of query creation much easier. Due to the special nature of data the query language can include features for display of the results of a query in graphical mode. The user interface can be extended by using pointing devices for the selection of objects of interest. The analysis of the results from the query may give feedback for refinement of the queries and show the direction

of further investigation The language should be powerful enough to cover the number of algorithms and large variety of data types stored in spatial databases.

Multidimensional rule visualization: Discovering knowledge is not enough because it has to be presented in a manner that the user can understand easily. One of the most effective ways of digesting the rules discovered is through graphical visualizations. Humans are very good at interpreting visual data and scenes. This fact should be exploited in the data mining process. Multidimensional data visualization has been studied, but multidimensional rule visualization is still an immature area. Spatial data mining can use some well developed visualization techniques in computer graphics in this case.

#### 4. CONCLUSION

We have shown that spatial data mining is a promising field of research with wide applications in GIS, medical imaging, robot motion planning, etc. Although, the field is quite young, a number of algorithms and techniques have been proposed to discover various kinds of knowledge from spatial data. We surveyed existing methods for spatial data mining and mentioned their strengths and weaknesses. This led us to future directions and suggestions for the spatial data mining field in general. The variety of yet unexplored topics and problems makes knowledge discovery in spatial databases an attractive and challenging research field. We believe that some of the suggestions that we mentioned have already been thought about by researchers and work may have already started on them. But what we hope to achieve is to give the reader a general perspective of the field.

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